Name________________________________

Math 125 — Quiz 1 — 30 Minutes — 11:50-12:20, Tuesday, Oct. 3, 2017

(3 questions, 40 points, no notes or calculator permitted)

1. (10 points) Let A be the area above the x-axis below the curve $y = x^2$ between $x = 2$ and $x = 10$. Let $n = 4$. Compute

(a) the n-th left endpoint Riemann sum for A;

(b) the n-th right endpoint Riemann sum for A;

(c) the n-th trapezoid rule for A;

(d) in each case (a)–(c), find the difference

$$(\textbf{estimate}) - (\textbf{true value}).$$

In this case the true value is $\int_2^{10} x^2 dx = 330\frac{2}{3} = 330.\overline{6}$.

NOTE: No calculators are permitted, but the arithmetic by hand should take you no more than a few minutes.

2. (10 points) Let A denote the area of the part of the ellipse $4x^2 + y^2 = 100$ that is in the first quadrant.

(a) (3 points) Write a definite integral for A, that is, an expression of the form $\int_a^b f(x)dx$.

(b) (7 points) Write a limit of the form $\lim_{n\longrightarrow\infty} \sum_{i=1}^n$(SOMETHING) for the area A.

3. (20 points) In this problem the units for distance, velocity, and acceleration are respectively cm, cm/sec, and cm/sec^2. In class we looked at the case of a magnetic field rotating counterclockwise from the positive x-direction at 1 rad/sec, causing an acceleration function $a(t) = \sin(t)$ on an object that's constrained to move up and down in a narrow tube along the y-axis. In this problem suppose that everything's the same, except for two things: (1) the magnetic field rotates at 1 rev/sec (equivalent to 2π rad/sec) in the clockwise direction starting from the positive y-direction, causing an acceleration function $a(t) = \cos(2\pi t)$, and (2) the object still has initial velocity 0 but on the y-axis it starts from y_0, which is not necessarily zero.

(a) (15 points) Find formulas for the object's velocity $v(t)$ and its height $y(t)$. The initial position y_0 should appear in your formula for $y(t)$.

(b) (5 points) Suppose that you know that the object spends an equal amount of time above and below the origin. Find y_0.

Answers

1. (a) 240, (b) 432, (c) 336, (d) $-90\frac{2}{3}$, $101\frac{1}{3}$, $5\frac{1}{3}$ (so the trapezoid rule is much better)

2. (a) The ellipse crosses the x axis when $y = 0$, that is, when $x = \pm 5$, so the area of the part in the first quadrant is $\int_0^5 \sqrt{100 - 4x^2}dx$; (b) $\lim_{n \longrightarrow \infty} \sum_{i=1}^n \frac{5}{n}\sqrt{100 - 4(\frac{5i}{n})^2}$.

3. (a) $a(t) = \cos(2\pi t)$, $v(t) = \frac{1}{2\pi}\sin(2\pi t)$, $y(t) = y_0 + \frac{1}{4\pi^2} - \frac{1}{4\pi^2}\cos(2\pi t)$,
(c) Set $y_0 + \frac{1}{4\pi^2} = 0$, so that $y_0 = -\frac{1}{4\pi^2}$ cm.