

Name _____

Math 125

Quiz 1 — 30 Minutes

12:50-1:20, Tuesday, Oct. 3, 2017

(3 questions, 40 points, no notes or calculator permitted)

1. (10 points) Let A be the area above the x -axis below the curve $y = x^2 - 2x$ between $x = 2$ and $x = 10$. Let $n = 4$. Compute

- (a) the n -th left endpoint Riemann sum for A ;
- (b) the n -th right endpoint Riemann sum for A ;
- (c) the n -th trapezoid rule for A ;
- (d) in each case (a)–(c), find the difference

(estimate) – (true value).

In this case the true value is $\int_2^{10} (x^2 - 2x) dx = 234\frac{2}{3} = 234.\bar{6}$.

NOTE: No calculators are permitted, but the arithmetic by hand should take you no more than a few minutes.

2. (10 points) Let A denote the area of the part of the ellipse $25x^2 + y^2 = 100$ that is in the first quadrant.

- (a) (3 points) Write a definite integral for A , that is, an expression of the form $\int_a^b f(x) dx$.
- (b) (7 points) Write a limit of the form $\lim_{n \rightarrow \infty} \sum_{i=1}^n (\text{SOMETHING})$ for the area A .

3. (20 points) In this problem the units for distance, velocity, and acceleration are respectively cm, cm/sec, and cm/sec². In class we looked at the case of a magnetic field rotating counterclockwise from the positive x -direction at 1 rad/sec, causing an acceleration function $a(t) = \sin(t)$ on an object that's constrained to move up and down in a narrow tube along the y -axis. In this problem suppose that everything's the same, except for two things: (1) the magnetic field rotates at 5 rev/sec (equivalent to 10π rad/sec) in the clockwise direction starting from the positive y -direction, causing an acceleration function $a(t) = \cos(10\pi t)$, and (2) the object still has initial velocity 0 but on the y -axis it starts from y_0 , which is not necessarily zero.

- (a) (15 points) Find formulas for the object's velocity $v(t)$ and its height $y(t)$. The initial position y_0 should appear in your formula for $y(t)$.
- (b) (5 points) Suppose that you know that the object spends an equal amount of time above and below the origin. Find y_0 .

Answers

- (a) 160, (b) 320, (c) 240, (d) $-74\frac{2}{3}$, $85\frac{1}{3}$, $5\frac{1}{3}$ (so the trapezoid rule is much better)
- (a) The ellipse crosses the x axis when $y = 0$, that is, when $x = \pm 2$, so the area of the part in the first quadrant is $\int_0^2 \sqrt{100 - 25x^2} dx$; (b) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \sqrt{100 - 25(\frac{2i}{n})^2}$.
- (a) $a(t) = \cos(10\pi t)$, $v(t) = \frac{1}{10\pi} \sin(10\pi t)$, $y(t) = y_0 + \frac{1}{100\pi^2} - \frac{1}{100\pi^2} \cos(10\pi t)$,
(c) Set $y_0 + \frac{1}{100\pi^2} = 0$, so that $y_0 = -\frac{1}{100\pi^2}$ cm.